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THE PROPERTIES OF THE VISUAL EXCITATION-CURVES

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A graph which is built upon the explicit assumption that a certain quantity is to be held constant can convey no information about any changes which presuppose that this quantity is also variable. Many studies which have been made of vision often seek to extract information from a graph respecting some property which had already been definitely assumed to be constant. A function such as

$$V = 32t$$

describes how a body falls. This function is linear and one can see readily that the area under this curve, between given limits, embodies or represents the total distance traveled by the falling body. The ordinate of this curve represents the instantaneous velocity of the object, while the slope of the curve embodies the acceleration of the object.

In the study of the phenomena of vision the fundamentally illuminating experiments are those of intensity discrimination and of acuity. In many of the early attempts to measure the ability of the eye to detect intensity changes and acuity changes one finds the effort made to extract information about intensity discrimination from graphs which were derived through keeping intensity constant. The converse is also often made, when persons seek information regarding frequency discrimination from experimental data gathered when frequency was held constant.

The excitation and the Elementarempfindungen curves are graphs which were worked out by taking certain arbitrarily chosen primaries. The ordinates represent the amounts or the numbers of the units of the chosen primaries; the area under the excitation curve will (and does) represent luminosity or, if one prefers, the number of functioning rods and cones while the slope of the curve depicts hue discrimination or the magnitude of the change in hue that a given eye can detect. In the case of the excitation curve we can readily interpret both the instantaneous value of the function, and the area underneath the curve and also the slope of the curve in terms of visual discriminations, just as readily as the physicist can assign physical meanings to the corresponding properties of the curve depicting the behavior of a falling body.

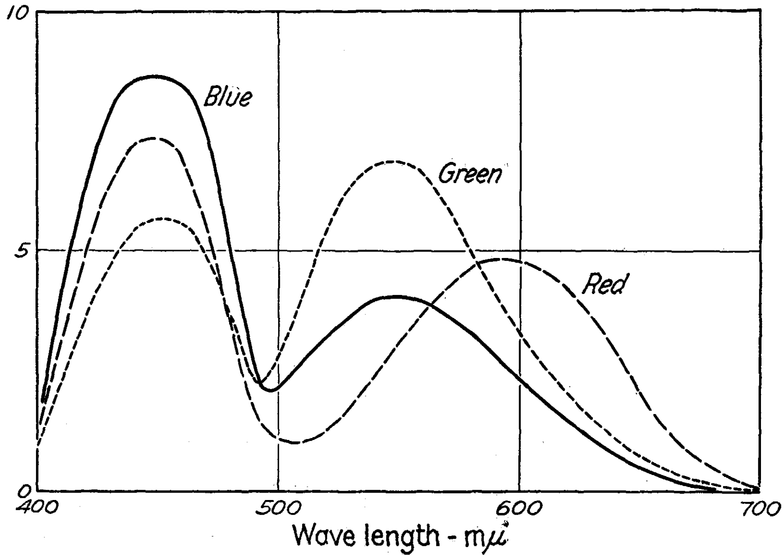


Fig. 1. Helmholtz's three fundamental primaries.

In the study of vision the following facts are explainable if the axis of abscissae be variable wave length: (1) color-blindness; (2) luminosity or brightness; (3) color-mixture phenomena; (4) hue-sensitivity; (5) hue-discrimination; (6) white complementary color pairs; (8) saturation; and (9) minimal hue detection. The facts are explainable without bringing in anything regarding a variation of intensity as will be seen later and much more fully in subsequent studies. The visual psychological properties that are interpretable in terms of varying intensity of the light stimulus constitute another distinct group. In explaining the above group we keep the intensity constant and vary only the wave length.

As the excitation curves are given us by Helmholtz (1) (see Fig. 1) and many others, it is impossible to read off from them the nine psychological characteristics mentioned above. Hecht (2) recognizes that it is possible to apply a simple homogeneous linear transformation to these historic excitation curves and so transform them that they will give the above-mentioned items of information regarding the nature of the visual process. Since these excitation curves are not the direct product of experimental data, a certain non-uniqueness belongs to them as the result of a mathematical operation upon other functions which *were experimentally determined.* Those who set up the excitation curve for us were not restricted in their

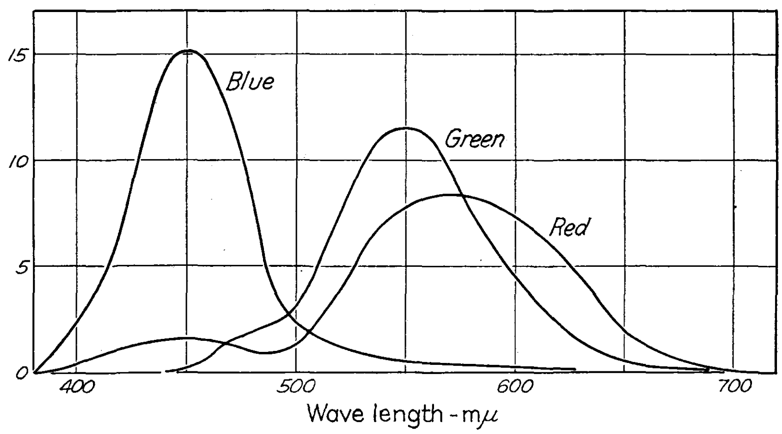


Fig. 2. The curves as determined from the data of Wright.

choice to any particular set of three primaries. If, for example, one takes the curves of Wright (3) (see Fig. 2), he can apply his mathematical technique of transformation to an infinite number of, sets of three, assumed or pseudo-primaries. There is only one restriction which operates to prevent the absolute freedom of choice of the three new pseudo-primaries. This can be illustrated thus: Suppose that *Wright* took B, G, and R to be his primaries and that the *reader* chose to take colors x, y, and z to be his primaries where x, y, and z are thus defined:

$$x = a_1 B + a_2 G + a_3 R$$
$$y = b_1 B + b_2 G + b_3 R$$
$$z = c_1 B + c_2 G + c_3 R$$

In this transformation the new primaries are *functions,* (not arithmetical numbers), of the red, green and blue that were primaries, for example, as postulated by Wright. One can again state the new or pseudo-primaries as connected up with still other primaries as:

$$u = l_1\,x + l_2\,y + l_3\,z$$
$$v = m_1\,x + m_2\,y + m_3\,z$$
$$w = n_1\,x + n_2\,y + n_3\,z$$

Such transformations show that any set of three colors can be taken as primary provided that in making the transformation the determinant in the denominator does not vanish.

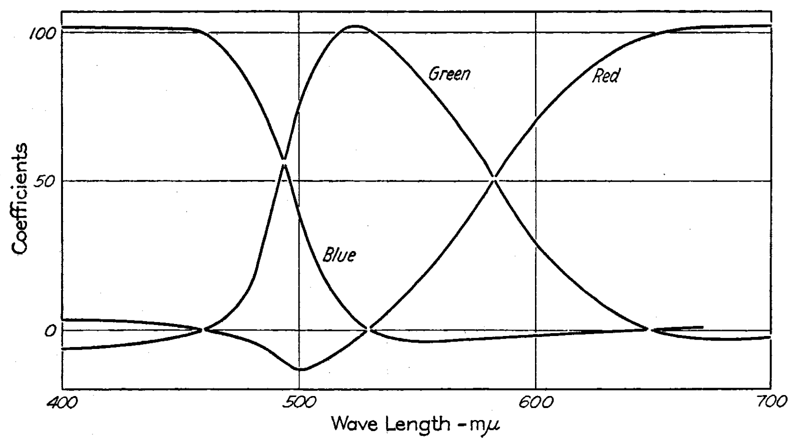


Fig. 3. The curves of Koenig and Dieterici.

Suppose that we solve for B in the set of equations just given,

$$B = \frac{\begin{vmatrix} x & a_2 & a_3 \\ y & b_2 & b_3 \\ z & c_2 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}$$

If the determinant in the denominator vanishes, then we find that we selected as a primary, a color which cannot serve as such in this set of equations got from the original experimental curves. In the function, which defines x, y, and z the symbol

B = a certain unit amount of blue,
G = a certain unit amount of green,
R = a certain unit amount of red.

The primary x, where, to be specific, x is say $\lambda = 500$ m μ of the given spectrum, might be thus represented,

$$x_{500} = 50R + 250G + 75B,$$

that is, it takes 50 units of R plus 250 units of G and 75 units of blue to match the color of $\lambda = 500$ of the spectrum which is being matched. Similar equations would tell how to duplicate the remaining two primaries, namely, y and z.

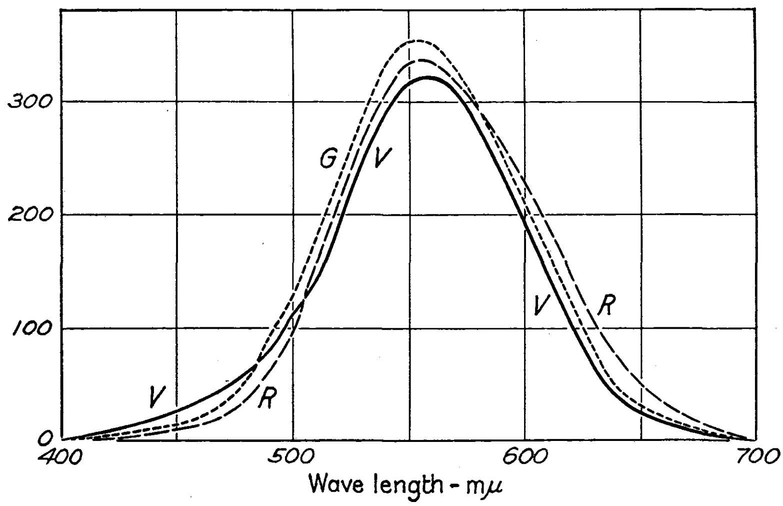


Fig. 4. Hecht's transformed excitation curve.

Again suppose that one knew both that 450 m μ = 102B + 2G + 4R, and that

$$B = [(a_1)x + (a_2)y + (a_3)z]$$
$$G = [(b_1)x + (b_2)y + (b_3)z]$$
$$R = [(c_1)x + (c_2)y + (c_3)z]$$

it then follows that

$$450 \text{ m } \mu = 102\,[(a_1 + b_1 + c_1)x] + 2\,[(a_2 + b_2 + c_2)y] - 4\,[(a_3 + b_3 + c_3)z].$$

Thus one demonstrates that it is possible to get an infinite number of sets of equations of x, y, and z. The student must, therefore, be alert to note whether or not the equation describes the primary of the experimental curves or of the transformed curves. By making such a linear homogeneous transformation the student of vision can move backwards or forwards from any set of three (primaries) to any other.

EXCITATION CURVES

These excitation or sensation curves were first obtained experimentally through mixing colors. Some spectrum was taken as standard and then through a process of matching, using only three different colors, this entire spectrum was duplicated. This process can readily be sensed by the reader from Fig. 5. Here is a three-fold source of light and it is easy to project, upon a screen, such amounts of the three colors as will duplicate any desired part of the spectrum assumed as standard.

Since these curves are the graphs of data embodying color mixtures which occur when the spectrum which we used was

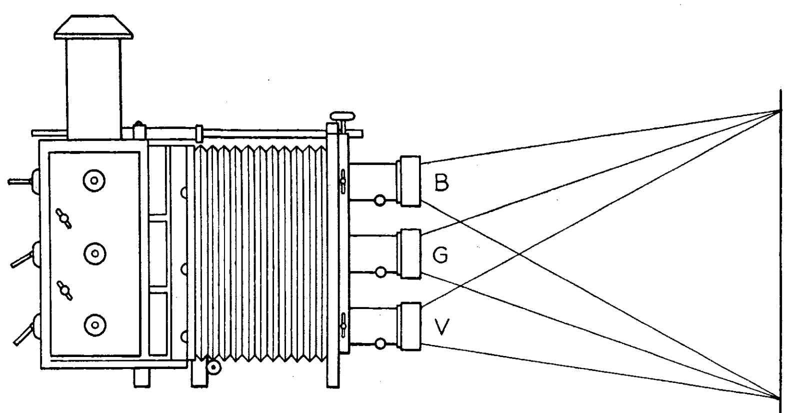


Fig. 5. A method of mixing colors so as to duplicate a standard spectrum.

a constant energy spectrum or the equivalent, it follows that these three sensation curves do not include variations of intensity. In a preceding section we showed that intensity cannot be represented (save as it is a constant) by means of these curves. Attempts have been made, by some who had not reasoned out very thoroughly the facts in the case, to introduce brightness into these curves. Hecht points out two such attempts (2).

One of these attempts rested upon the assumption that there are three groups of receptors in the retina, but there are not the same number of receptors in each group. Experiments by Judd (4), and others (5) pointed out the fact that some colors are very much more effective in aiding in producing white than others. For example, it takes a great deal more yellow to cancel violet than it does of wave length 576 m μ to cancel 475 m μ and again the relative amounts here are very different from what they are for 609 m μ and 494 m μ. The result of these experiments was taken to mean that the population of the cones is as 1 : 75 : 100. There are said to be 75 green-producing cones and 100 red-producing cones to every one of blue. The other attempt to incorporate brightness into these sensation curves made the assumption that the number of cones of each family is the same as the number of the other two families and that at any particular threshold of stimulation the functioning receptors are distributed among the three families and that in each case where the psychological experience is an experience of white that the cones are functioning in the ratio of 1 blue to 75 green, and to 100 red. Both these attempts to read luminosity or brightness into the excitation curves involves a serious weakness. Let us consider the latter expedient first. This says that there are the same number of cones of each family, but that one sees white when they are excited in the ratio of 1 : 75 : 100. Let us take a few examples here to show how the flaw in this assumption can be made manifest.

Blue	*Green*	*Red*	*Experience of the Individual*
1	75	100	Sees white
10	750	1000	Sees white
14	1000	1000	Does not see white
600	1000	1000	Sees blue

This table can be extended indefinitely. If the individual sees only white when the stimulated cones are in the ratio of 1 : 75 : 100, and if there are equal numbers of cones of each family, it follows, since the red cones are brought into play a hundred times as fast as the blue, that the former will all be called upon to function long before all the blue and green cones are. From that point on until the last cone threshold of the blue shall have been reached, no more red cones can be set into action and as the result of this, from here on there must be a color which is the result of the interaction of the green and the blue. Again, as there are 75 green cones called into action for every one blue cone, it follows again that, as we increase the intensity of the light, all the green cones will have been set into action long before all the blue cones are functioning. When such intensity of light has been reached which has set not only all the green cones but also all the red cones into action, then with all additional augmentations of the intensity of the light there must be experienced a blue color. This is inevitable because of the predominance of the blue cones activated as over against what is called for by the proportion 1 : 75 : 100.

In this way it is clear that if one would take white light and stimulate the retina, the subject would, at first, see white; then as the light is further increased there will be experienced a hue which is the resultant of the operation of the green and blue cones, as soon as all the red cones will have been set into action. Further increasing of the intensity of the stimulus will be accompanied by an experience of the colors of the violet end of the spectrum as soon as the green cones have been set into action.

This attempt to have the excitation curves incorporate the data of brightness very evidently involves the presence of a fallacy, for the simple reason that as we take white light and stimulate the retina with it we experience white or gray all the way up from light of lowest intensity to light of maximum intensity. Thus this theory that there are equal numbers of cones of each family and that the experience of white is the correlate of the action of these in the ratio 1 : 75 : 100 breaks down. If an individual experiences white all the way up from light of minimal intensity to light of maximal intensity it cannot follow that the population of the cones and their action are as this assumption postulates.

The second assumption still rests on the fact revealed by experiment to the effect that the relative efficacy of the colors in producing white is as 1 : 75 : 100. But how incorporate this property of brilliance into our curves which have been made only to express color-mixture data?

The situation so far is that those who gave us our excitation curves had a multiplicity of units (three). They had a unit of V, another unit of G and still another unit of R. What they then wished to do was to replace these three sets of units by a single unit. The lay of the land at the time can be shown thus. Three possibilities are open to the student of vision. They are: (a) one unit of V, one unit of G, one unit of R; or (b) a common unit of energy; or (c) a unit of brightness. Some writers continued to stumble along seeking to clear up the problems of vision by utilizing the threefold set of units. While they continued to do this they were unable to introduce into the curves anything more than what they had put into them; namely, color mixture data. Soon numerous attempts were made to introduce or to incorporate other facts into them. Brightness was chosen as one to be incorporated into the curves. The fact that brightness is a property of both "hue vision" and "white light vision" constituted it an outstanding trait of all visual experiences. In the dimmest light and throughout the entire range of moderate intensities as well as at maximal intensities, brightness is an ubiquitous trait. Perhaps it was for some such reason that students early attempted to incorporate this property into their curves. We have already seen that one attempt was made which proved premature. The second attempt made seems to be much more hopeful of meeting with success.

This attempt was made by those who were aware of the experimental findings which showed that violet was much more effective in producing white, when in combination with the other colors, than was either green or red. Carefully made experiments showed that this effectivity for producing whiteness of the various cone groups was as 1 : 75 : 100. One violet cone is as effective in giving white (or gray) as are 75 green cones, or as effective as 100 white cones.

Suppose then that these men assume that the population of the cone groups in the retina is *not* equal in number. This was taken for granted by those who made the preceding assumption. Let µs, therefore, accept the ratio 1 : 75 : 100. This seems to be a very reasonable hypothesis. Since one blue or violet cone is as effective as 75 green cones in production of white, then, what is more natural than that there should be 75 times as many green cones as there are violet cones? Likewise, since a violet cone is as effective, in the production of white, as 100 cones of the red producing group, what could be more plausible than to say that in the retina there are one hundred times as many red producing cones as violet producing. On this assumption hue would result from a stimulation of such a magnitude and of such a character as not to stimulate the cones in the proportion of 1V : 75G : 100R. Any stimulation which sets up an imbalance, from the standpoint of this ratio, would produce an experience of hue, and similarly any stimulation, no matter what its intensity (within the limits of the eye's sensitivity), which stimulated the cones in this ration of 1 : 75 : 100, would give rise to the experience of gray or white. Surely here is an assumption that one is prone to say must be adequate.

Now what about the hypothesis that the total population of the cones of the three groups or families is in the proportion indicated by the experiments on the relative efficacy of the different cones in producing the experience of white? Viewing the matter in an *a priori* deductive fashion one feels confident that the assumption is adequate.

Suppose now we get a red as saturated as we can and then increase its intensity. We can make an intensity discrimination experiment on this red and keep a record of the data. Finally we reach the maximum intensity. No brighter red can be discriminated by the eye than that which results when the light is of such intensity as to set all available cones, of the red family, into action. The brightest red we can discriminate is one which is brought about through the activation of all the cones of this particular family. Another experiment of the same kind can be made with green light and with a similar result and implying the same sort of interpretation. Let us now take a violet or blue and run this through the same type of experiment. If our assumption as to the relative distribution of cones were true, we would be unable to get anything like so many intensity discrimination steps when working with the violet as we do with the other two primaries. If we have only one violet cone to each 100 of the red family cones we could never get so bright a blue as we can a red or green. Likewise in the case of the green. Since brightness is shown to be proportional to the number of functioning retinal elements we could never have a green so bright as the maximum red and by far more would our maximum brightness of the blue fall short.

Experiment contradicts this. We can have a blue as bright as our brightest green or even our brightest red. So falls by the wayside this second attempt at incorporating brightness into the excitation curves. A new attempt remains to be made. It is a very simple matter to contrive a theory of vision that will explain a few detached facts, but what the scientist seeks is a theory that will simply embody or display all that is experimentally gleaned in the laboratory.

Hecht has developed for us a transformed set of excitation curves (2) (see Fig. 4). These were derived originally from color-mixture experiments and the data so gathered were subjected to a linear homogeneous transformation. This was, of course, the way that both the elementary and the fundamental excitation curves were obtained. The transformation that gave the above figure was the one from which Hecht proposes to deduce the properties of vision that are correlated with a change in wave length and with intensity kept constant.

THE NORMAL EYE

If these excitation-curves are to explain all the parts of vision under the restriction that the intensity of the stimulus is held constant, certain requirements with reference to the curves are immediately apparent.

(1) They are to represent three equi-numbered groups or families of receptors. To do this the functions are so transformed that the area under any curve is equal to the area of each of the other two.

(2) The demand is also made by the one setting up these transformed functions that the summed area under all three of these curves taken separately shall add up to give the observed visibility curve of the spectrum. Stated otherwise, this is the same as saying that the *sum* of the area under each of these three curves shall give the observed or experimentally checked visibility curve. It follows, also, incidentally that the total area under the curve which is the sum of the three separate excitation curves will represent the total number of functioning receptors when the *entire* spectrum is applied, and not some monochromatic light.

This second requisite will immediately cast out the experimental curves of Wright (see Fig. 2) as well as the excitation curves of Helmholtz (see Fig. 1) and other transformed functions such as those of Koenig. The visibility curve is a smooth continuous function having but one maximal value. Such a curve can not be got from the excitation curves of either Wright or Helmholtz. If one adds the ordinates of the Helmholtz excitation curves he will find that the resultant curve will have two maximal values and not one as is the case with the visibility curve. Since this is the state of affairs it follows that the elementary and the fundamental curves will not prove adequate to the task of explaining the visual process as a whole.

(3) These curves that we accept must be consistent with the observed hue experience. That is, these curves must tell something about the experience of different colors and also

about the *how much* of the experience. Later on it will be shown that this information will be imparted by adjusting the relative ordinates or heights of the excitation curves.

(4) These curves must also be consistent with hue discrimination. This item of the visual process will be inserted in the *slopes* of the curves. The reader will remember that in the case of the falling body the slope was constant and he will also persistently bear in mind that in this case he has three curves to grapple with, and not just one simple linear function, as in the case of the law of the falling body. *Hue* discrimination is, therefore, to be determined from the slopes of our three excitation curves, whereas observed hue is to be interpreted by the *relative* heights of the curves at any given wave lenght.

(5) These curves are constructed to describe the normal eye, and they must therefore not be expected to tell all about the vision of the abnormal eye, but there are several things that have been observed about the color blind which suggest the possibility of making significant minor adjustments in the excitation curves describing the normal eye.

(6) Our curves must also embody information regarding white complementary colors. This matter of color complementariness gives another "look in" at color mixture with the result that we find embodied in this phase of the problem of vision that several of the laws of algebra hold and that the numbers involved are of the character of complex numbers involving a real and an imaginary component.

(7) Once one has detected the real nature of what is at work in the experience of white complementariness he is literally forced into the making of a quantitative study of the phenomenon of saturation. Again if the three excitation curves we have settled upon are to be suitable for an adequate theory of vision we must be able to read the facts regarding saturation directly from our curves.

The above seven requirements are made of any set of curves which is to describe our visual experience. Will the three curves of Hecht meet these demands? Evidence of their adequacy is here given in the form of a quantitative analysis of hue discrimination.

AN APPLICATION OF THE VISUAL EXCITATION CURVES

The function $V = (32)t$ which describes the behavior of a falling body, displays three distinct bits of information: the instantaneous value of the velocity; the total distance traveled, and the acceleration. In this case the acceleration is zero.

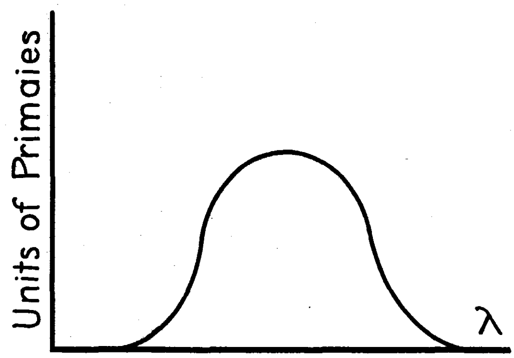
In the problem we are concerned with, we have not one but three curves with which to deal, and it is evident that we have a right to expect them to impart much more detailed information than could possibly be given by a single function. In the case of the function

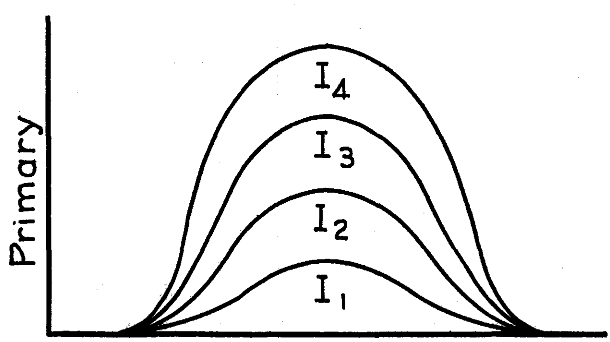
$$V = 32t$$

we have a constant slope, whereas the excitation functions have varying slopes throughout and further significant properties emerge as a consequence of the fact that they intersect at different points (5). The relative differences of the ordinates of these functions, corresponding to a given wave-length, are quantities which describe the most variable and individual property of vision.

The total area under the three curves (see Fig. 4) is a measure of the total number of functioning retinal elements (2). The area enclosed under the "visibility" curve expresses this number under the condition that prevails when the eye is stimulated by the entire spectrum, while the ordinate is a measure of color mixture, luminosity or of the number of receptors functioning in response to monochromatic light. The *relative* differences between the ordinates corresponding to a particular wave length is found, both through experiments and mathematical description, to be an extremely accurate measure of the experience of hue. The slopes of the curves in turn give us an excellent measure of hue discrimination. These properties are compactly given thus: $A \equiv$ area $\equiv$ number of activated receptors; $\frac{dA}{d\lambda} \equiv O \equiv$ ordinate $\equiv$ luminosity, etc.; $D \equiv$ relative difference of ordinates for a corresponding wave length; $\equiv$ hue sensitivity; and $\frac{d^2A}{d\lambda^2} \equiv$ slope $\equiv$ hue discrimination.

Perhaps one of the most telling arguments in support of the three-receptor theory of color vision is brought out clearly by means of these excitation curves. On an average there is a decided constancy of both "A" and $\frac{dA}{d\lambda}$ as shown by experi-





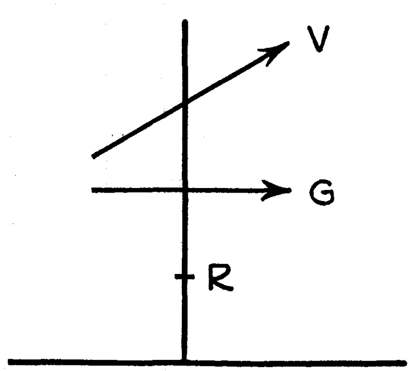


Fig. 6. (Upper). Fig. 7. (Center). Fig. 8. (Lower).

ments on different individuals. Taken in the large the element of individuality of the eye is shown progressively as one passes from the study of A, to the study of $\frac{dA}{d\lambda}$, and then to the study of $\frac{d^2A}{d\lambda^2}$. In life we recognize each other as individuals and accord to each person a uniqueness which makes him an unsubstitutable object. Science washes its hands of the attempt to get the unique, so say many scientists. Yet all know that as the order of the differential equation advances it calls for the positing of more and more initial or boundary conditions. Along just this way lies individuality. The more individual any affair is, the higher is the order of the differential equation. In our present problem we find that the only aspect or property of our curves which can be very greatly changed, relatively, and still leave the areas under the curve virtually unchanged is the slope of each of these curves. We can make a large change in the relative slopes of these curves at almost any point and yet keep our enclosed area a constant. In this respect we have a new light as to how these excitation curves may serve as a realistic account of the retinal physiological processes. The most variable feature about vision is the ability of the eye to make hue discriminations.

Since one of the most variable properties of the eye is its functioning in making hue discriminations it is certainly significant—and a telling blow for the three-element theory—that there is an aspect of the curves that lends itself to great modifications and yet, at the same time, leaves unaltered the total area under the curves. If we had but one curve it would be utterly impossible to do this. However, since we have three curves, each enclosing equal areas, and since these are placed as they are it is perfectly simple to change their relative slopes greatly without affecting the total area enclosed. This variability of the slopes of the three curves then is a trait that can be easily varied, its psychological correlate is given us by the fact that hue discrimination is equally variable from eye to eye. To fit our excitation curves for one eye and then for another, we are called upon only to vary the relative slopes of these curves. We can easily strike a general average that holds for many eyes in terms of A and $\frac{dA}{d\lambda}$. This is but a way of saying that these two quantities are fairly impersonal. It is not so easy to do this with the quantity $\frac{d^2A}{d\lambda^2}$. This quantity tells us something that is quite personal about the eye. It is not evident how we could get this high individual variability characteristic of hue discrimination if we did not have a multiplicity of excitation curves. Again, we say that this is one of the most significant arguments for the adequacy of the Young-Helmholtz theory.

The problem now before us is to check hue discrimination and to say just how it shall be given to us by the curves. (See Fig. 6.) At the two extreme ends the slope of the curve shown in Fig. 6 changes very slowly—hue discrimination is very poor at those places. At the top the slope is quite flat, almost horizontal for some distance; here also the discrimination of hues is very poor. If we were to vary the intensity I of the stimulus we would find that the curves would be of this general character. (See Fig. 7.) The greater the intensity the flatter the curve would be at the top.

Experimentally what must be done in carrying on a measurement of hue discrimination is to keep the hue intensity, $L_H(\lambda)$ = constant and make the change in the hue. If we could not do this our measurements would be meaningless. A table may be helpful, at this point.

λ_3	V	G	R	L_H	L_W	L_t
500	40	30	25	20	75	95
550	45	30	20	35	60	95
330	43	27	25	20	75	95
	48	27	20	35	60	95

In making readings the student must exercise caution in noting which curve is the lowest in the region where he is taking the measurements. The balanced part of the visual effect is always three times the value of the least ordinate at the wave-length considered. Let us take the cases where

$$\begin{Bmatrix} V_1 = 48 \\ G_1 = 27 \\ R_1 = 20 \end{Bmatrix} \text{ and } \begin{Bmatrix} V_2 = 45 \\ G_2 = 30 \\ R_2 = 20 \end{Bmatrix}$$

$$\triangle H = (V_1 - V_2) + (G_1 + G_2) = (48\text{–}45) + (27 + 30)$$

This expression gives us the rates of change of the ordinates for V and for G. The reason that the ordinate of the red curve does not appear is that care was taken to keep the luminosity constant. To be more accurate what is kept constant is the white luminosity. Having found the relative differences of the two biggest components for any given lambda and having equated this to H, we may now get the rate of change of hue with respect to wave length by dividing the above equation by $\triangle\lambda$.

There results:

$$\frac{\Delta H}{\Delta\lambda} = \frac{V_1 - V_2}{\Delta\lambda} - \frac{G_1 - G_2}{\Delta\lambda}$$

$$\therefore \frac{dH}{d\lambda} = \frac{V_1 - V_2}{(d\lambda)} - \left(\frac{G_1 - G_2}{d\lambda}\right)$$

to the right of the point where we have the first intersection of the lowest curve the variable which will disappear from the equation is V. Thus the measure of hue discrimination for all wave-lengths to the *right* of the point where the Red and Violet intersect will not involve V. The expression for hue discrimination throughout this interval is given by

$$\frac{\Delta H}{\Delta\lambda} = \frac{R_1 - R_2}{\Delta\lambda} - \left(\frac{G_1 - G_2}{\Delta\lambda}\right)$$

$$\frac{dH}{\Delta\lambda} = \frac{dR}{d\lambda} - \frac{dG}{d\lambda}$$

This information can be shown graphically in a way that may help the reader to a better appreciation of how hue discrimination is measured. Let us take a particular ordinate and let points on it indicate the points where it cuts the excitation curves. (See Fig. 8.) At the points V and G are indicated the slopes of the V and G. curves. If these indicating lines are diverging, converging or parallel the hue discrimination will be an increasing, decreasing or a constant function.

From the curves that have been finally accepted as giving a description of the facts of vision it is easy to show from the equations below,

$$\frac{dH_1}{d\lambda} = \frac{d}{d\lambda}(V_\lambda - G_\lambda)$$

$$\frac{dH_2}{d(\lambda)} = \frac{d}{d\lambda}(R_\lambda - G_\lambda)$$

the right hand side of each equation to be a function of lambda. From this we have that

$$\frac{dH_{(\lambda)}}{d_{(\lambda)}} = f\ (\lambda)$$

On this assumption we may say that

$$\frac{\Delta H_{(\lambda)}}{\Delta_{(\lambda)}} \approx f\ (\lambda)$$

Here we take a certain finite increment of $H_{(\lambda)}$ and assume it to be constant and at the same time *equal to the least perceptible change* in hue. Since $\Delta H_{(\lambda)}$ = constant, it follows that constant $\approx f\ (\lambda) \cdot \Delta(\lambda)$.

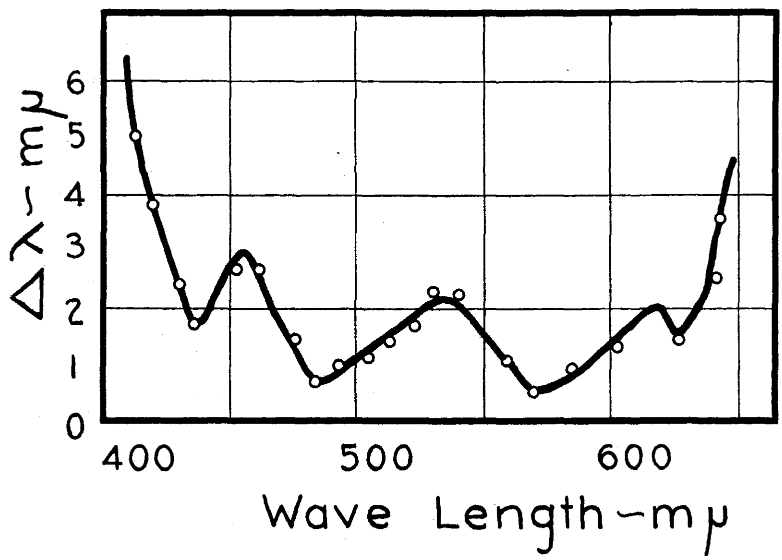


Fig. 9. The data of hue discrimination of Lauren's eye taken from Laurens and Hamilton '29. The smooth curve gives the data, whereas the points are computed from the primaries V, G, and R.

It remains, therefore, only to plot the function between $\Delta\lambda$ and λ and then to check this curve with the data from experiments. (See Fig. 9.)

The curve here shows a remarkable correspondence between the facts revealed by experiments and the calculated curve is sufficient proof that herein is offered a satisfactory method of measuring hue discrimination.

REFERENCES

1. **H. von Helmholtz.** Handbuch der Physiologischen Optik. Hamburg und Leipzig, 1896, Vol. XIX, 1334 p.
2. **S. Hecht.** The development of Thomas Young's theory of color vision. J. Opt. Soc. Amer., Vol. 20, 1930, pp. 231–270.
3. **W. D. Wright.** A re-determination of the mixture curves of the spectrum. Trans. Opt. Soc., Vol. 31, 1929–30, pp. 201–218.
4. **D. B. Judd.** The mixture data embodied in the tentative curves of Hecht's theory of vision. J. Opt. Soc. Amer., Vol. 20, 1930, pp. 647–660.
   Comparison of Wright's data on equivalent color stimuli with the O. S. A. data. Ibid., Vol. 21, 1931, pp. 699–728.
   Chromaticity sensibility to stimulus differences. Ibid., Vol. 22, 1932, pp. 72–108.
5. **Frank Allen.** On the discovery of four transition points in the spectrum and the primary colour sensations. Philos. Mag., Vol. 38, 1919, pp. 55–81.
   On reflex visual sensations. Amer. J. Physiol. Optics, Vol. 5, 1924, pp. 341–375.
   On reflex visual sensations and color contrast. Ibid., Vol. 6, 1925, pp. 339–374.